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HOLOMORPHIC FUNCTIONS WHICH ARE HIGHLY NONINTEGRABLE AT THE BOUNDARY

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ABSTRACT

Given a bounded convex domain D in $\mathbb{C}^N$ with smooth boundary and a positive continuous function φ on D, it is proved that there is a holomorphic function f on D such that $|f|\varphi$ is nonintegrable on $M \cap D$ whenever M is a real submanifold of a neighbourhood of a point of bD which intersects bD transversely.

1. The result

In a recent paper [J] P. Jakobczak showed that given a bounded convex domain $D \subset \mathbb{C}^N$ with smooth boundary there is a function f holomorphic on D such that $\int_{M\cap D} |f| dS = +\infty$ for every complex submanifold M of a neighbourhood of $\overline{D}$ which intersects bD transversely, where dS is the surface area measure.

In the present note we show that there are holomorphic functions on D with more singular nonintegrability behavior at the boundary:

THEOREM 1.1: *Let $D \subset \mathbb{C}^N$ be a bounded convex domain with boundary of class $\mathcal{C}^1$ and let φ be a positive continuous function on D. There is a holomorphic function f on D with the following property: Let $z \in bD$, let $U \in \mathbb{C}^N$ be an open neighbourhood of z, let M be a real submanifold of U of class $\mathcal{C}^1$ which meets bD at z transversely, and let dS be the surface area measure on M. Then*

$$(1.1) \qquad \int_{M\cap D} |f|\varphi dS = +\infty.$$

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2. The function

Suppose that $S \subset \mathbb{C}^N$ is a set such that there is a unique real hyperplane H containing S and assume that H does not contain the origin. Given $\delta > 0$ we denote by $T(S,\delta)$ the union of translates of S in the direction perpendicular to H away from the origin for a distance τ, $0 \le \tau \le \delta$, that is

$$T(S,\delta) = \bigcup_{0\le\tau\le\delta} (\tau\mathbf{n} + S),$$

where $\mathbf{n}$ is the unit vector perpendicular to H pointing into the direction of the component of $\mathbb{C}^N \setminus H$ that does not contain the origin.

Suppose that $D \subset \mathbb{C}^N$ is a bounded convex domain with smooth boundary. With no loss of generality assume that $0 \in D$. Let E_j be a sequence of compact polyhedral bodies,

$$0 \in \text{Int } E_1 \subset\subset \text{Int } E_2 \subset\subset \cdots \subset \bigcup_{j=1}^{\infty} E_j = D.$$

LEMMA 2.1: *For each $j \in \mathbb{N}$ there are $\delta_j > 0$, $n_j \in \mathbb{N}$, compact polyhedral bodies P_{ji}, $1 \le i \le n_j$, $P_{j0} = E_j$, $P_{j,n_j+1} = E_{j+1}$, satisfying*

$$\text{Int}\, P_{j0} \subset\subset \text{Int}\, P_{j1} \subset\subset \cdots \subset\subset \text{Int}\, P_{j,n_j+1},$$

and for each i, $1 \le i \le n_j$, a closed $(2N-1)$-dimensional face F_{ji} of P_{ji} such that

$$T(F_{ji},\delta_j) \subset \text{Int}\, P_{j,i+1} \quad (1 \le i \le n_j) \tag{2.1}$$

and such that if $T_j = \bigcup_{i=1}^{n_j} T(F_{ji},\delta_j)$, then given a C^1 arc γ: $[0,1] \to \mathbb{C}^N$, $\gamma([0,1)) \subset D$, $\gamma(1) \in bD$, such that $\gamma'(1)$ is not tangent to bD at $\gamma(1)$ there are a neighbourhood W of γ in the C^1 topology of C^1 maps from $[0,1]$ to $\mathbb{C}^N$ such that if $\lambda \in W$ is an arc, $\lambda([0,1)) \subset D$, $\lambda(1) \in bD$, then for each $j \ge j_0$, $\lambda([0,1)) \cap T_j$ contains an arc whose length is at least δ_j.

As we shall see it will be essential that one can use the same j_0 for all arcs sufficiently close to γ. It should cause no confusion that we are using the word arc for injective continuous maps and also for their images.

Suppose that we have already proved Lemma 2.1.

LEMMA 2.2: *Let T_j, $j \in \mathbb{N}$, be as in Lemma 2.1. Given a sequence M_j of positive numbers increasing to $+\infty$ there is a function f holomorphic on D such that* $\mathrm{Re}\, f(z) > M_j$ $(z \in T_j)$ *for each* $j \in \mathbb{N}$.

Proof: Denote by $< | >$ the Hermitian inner product on $\mathbb{C}^N$. Fix $j \in \mathbb{N}$ and i, $1 \le i \le n_j$, and let $0 < M < \infty$ and $\varepsilon > 0$. Let H be the real hyperplane containing F_{ij}. Since $0 \notin H$ it follows that there are a unit vector $\mathbf{n} \in \mathbb{C}^N$ and $\lambda > 0$ such that

$$H = \{z \in \mathbb{C}^N \colon \mathrm{Re} < z|\mathbf{n} >= \lambda\}.$$

Write $\psi(z) =< z|\mathbf{n} >$. By the properties of F_{ji} and $P_{j,i-1}$, $\psi(T(F_{ji}, \delta_j))$ is a compact set contained in $\{w \in \mathbb{C}\colon \mathrm{Re}\, w \ge \lambda\}$ and $\psi(P_{j,i-1})$ is a compact set contained in $\{w \in \mathbb{C}\colon \mathrm{Re}\, w < \lambda\}$. The one-variable Runge theorem gives a polynomial p such that $|p| < \varepsilon$ on $\psi(P_{j,i-1})$ and $\mathrm{Re}\, p > M$ on $\psi(T(F_{ji}, \delta_j))$, so $Q = p \circ \psi$ is a complex valued polynomial on $\mathbb{C}^N$ such that

(i) $|Q| < \varepsilon$ on $P_{j,i-1}$,

(ii) $\mathrm{Re}\, Q > M$ on $T(F_{ji}, \delta_j)$.

As in [GS1, p. 433] use the preceding fact and (2.1) and perform the induction with respect to i, $1 \le i \le n_j$, to prove that given $M < \infty$ and $\varepsilon > 0$ there is a complex valued polynomial Q such that

$$|Q| < \varepsilon \quad \text{on } E_j, \qquad \mathrm{Re}\, Q > M \quad \text{on } T_j.$$

Reasoning in the same way and performing the induction with respect to j we complete the proof. ∎

3. Proof of Theorem 1.1, assuming Lemma 2.1

Let φ be a positive continuous function on D and let T_j, δ_j, $j \in \mathbb{N}$, be as in Lemma 2.1. Since each T_j is compact it follows that $\inf\{\varphi(z)\colon z \in T_j\} > 0$, so by Lemma 2.2 there is a function f holomorphic on D such that

$$\delta_j \varphi(z)\, \mathrm{Re}\, f(z) \ge 1 \quad (z \in T_j). \tag{3.1}$$

Let $\gamma\colon [0,1] \to \mathbb{C}^N$ be a smooth arc, $\gamma([0,1)) \subset D$, $\gamma(1) \in bD$, $\gamma'(1)$ not tangent to bD at $\gamma(1)$. By Lemma 2.1 there are a $\mathcal{C}^1$neighbourhood W of γ and $j_0 \in \mathbb{N}$ such that for each arc $\lambda \in W$, $\lambda([0,1)) \subset D$, $\lambda(1) \in bD$, and each $j \ge j_0$, the set $\lambda([0,1)) \cap T_j$ contains an arc β_j whose length is at least δ_j. By (3.1) it follows that $\int_{\beta_j} \varphi \max(\mathrm{Re}\, f, 0) ds \ge 1$ where ds is the arclength. It follows that for all such λ,

$$\int_{\lambda([0,1)) \cap E_j} \varphi \max(\mathrm{Re}\, f, 0) ds \ge j - j_0 \quad (j \ge j_0). \tag{3.2}$$

To prove Theorem 1.1 write $\Phi = \varphi \max(\operatorname{Re} f, 0)$ and assume that $z \in bD$, that U is an open neighbourhood of z and that M is a real submanifold of U which intersects bD at z transversely. Let $m = \dim M$. Clearly $1 \leq m \leq 2N$. If $m = 1$ then dS is the arclength and so $\int_{M\cap D} \Phi dS = +\infty$ by the preceding discussion. So assume that $m \geq 2$. With no loss of generality assume that $z = 0$. Choose an orthonormal basis in $\mathbb{R}^{2N} = \mathbb{C}^N$ such that the first m coordinate axes $x_1, \ldots, x_m$ span the tangent space T to M at 0, and such that the coordinate axis x_m is transverse to bD at 0 with its positive direction pointing outside D.

Near 0, M is a graph over its tangent space T so we may assume that $U = U_1 \times U_2$, U_1 a neighbourhood of 0 in R^m, U_2 a neighbourhood of 0 in $\mathbb{R}^{2N-m}$ and that there are smooth real functions $\varphi_{m+1}, \ldots, \varphi_{2N}$ on U_1, $\varphi_j(0) = 0$, $(D\varphi_j)(0) = 0$ $(m+1 \leq j \leq 2N)$, such that

$$U \cap M = \{(x_1, \ldots, x_m, \varphi_{m+1}(x_1, \ldots, x_m), \ldots, \varphi_{2N}(x_1, \ldots, x_m)) \colon (x_1, \ldots, x_m) \in U_1\}.$$

By transversality, after shrinking U if necessary, we may assume that $U \cap M \cap bD$ is a submanifold of $M \cap U$ of real codimension 1. Since the coordinate axis x_m is transverse to bD at 0 it follows that the tangent space $T_{M\cap U\cap bD}(0)$ is a real hyperplane in T which can be written as a graph over

$$\{x_1, \ldots, x_{m-1}, 0, \ldots 0) \colon x_i \in \mathbb{R}, 1 \leq i \leq m-1\}$$

and consequently, after shrinking U if necessary, we may assume that $U_1 = U_1' \times U_1''$ where U_1' is a neighbourhood of 0 in $\mathbb{R}^{m-1}$, $U_1'' = (-r, r)$ for some $r > 0$, and that there are smooth functions $\psi_m, \ldots, \psi_{2N}$ on U_1' such that $U \cap M \cap bD = \{(x_1, \ldots, x_{m-1}, \psi_m(x_1, \ldots, x_{m-1}), \ldots, \psi_{2N}(x_1, \ldots, x_{m-1})) : (x_1, \ldots, x_{m-1}) \in U_1'\}$ where $\psi_j(0) = 0$ $(m \leq j \leq 2N)$. Since $U \cap M \cap bD \subset U \cap M$ it follows that $\psi_j(x_1, \ldots, x_{m-1}) \equiv \varphi_j(x_1, \ldots, x_{m-1}, \psi_m(x_1, \cdots, x_{m-1}))$ $(m+1 \leq j \leq 2N)$. Obviously,

$$\begin{aligned} U \cap M \cap D = \{&(x_1, \ldots, x_{m-1}, x_m, \varphi_{m+1}(x_1, \ldots, x_m), \ldots, \varphi_{2N}(x_1, \ldots x_m)) \colon \\ &(x_1, \ldots, x_{m-1}) \in U_1', -r < x_m < \psi_m(x_1, \ldots, x_{m-1})\}. \end{aligned}$$

The properties of E_j imply that, after passing to a smaller U_1' if necessary, there are j_0 and a sequence ε_j, $j \geq j_0$, decreasing to zero, such that if

$$\begin{aligned} M_j = \{&(x_1, \ldots, x_m, \varphi_{m+1}(x_1, \ldots, x_m), \ldots, \varphi_{2N}(x_1, \ldots, x_m)) \colon \\ &(x_1, \ldots, x_{m-1}) \in U_1', -r < x_m < \psi_m(x_1, \ldots, x_{m-1}) - \varepsilon_j\} \end{aligned}$$

then $M_j \subset M \cap U \cap E_j$ and $\bigcup_{j=j_0}^{\infty} M_j = M \cap U \cap D$.

Let $\gamma(t) = ((0,\ldots,0,t,\varphi_{m+1}(0,\ldots,0,t),\ldots,\varphi_{2N}(0,\ldots,0,t))$ $(-r < t < 0)$. Then $\gamma(t) \in D$ $(-r < t < 0)$ and $\gamma'(0) = (0,\ldots,0,1,0,\ldots,0)$ (with 1 at the mth entry) is not tangent to bD at $0 = \gamma(1)$.

The preceding discussion now implies that, after shrinking U_1' if necessary and passing to a larger j_0 if necessary, we may assume that if $\Lambda(x_1,\ldots,x_{m-1};j) = \{(x_1,\ldots,x_{m-1},\ t,\varphi_{m+1}(x_1,\ldots,x_{m-1},t),\ldots,\varphi_{2N}(x_1,\ldots,x_{m-1},t))\colon\ -r < t < \psi_m(x_1,\ldots,x_{m-1}) - \varepsilon_j\}$ then

$$\int_{\Lambda(x_1,\ldots,x_{m-1};j)} \Phi ds \geq j - j_0((x_1,\ldots,x_{m-1}) \in U_1', j \geq j_0),$$

where ds is the arclength, that is,

$$\int_{-r}^{\psi_m(x_1,\ldots,x_{m-1})-\varepsilon_j} \Phi(x_1,\ldots,x_{m-1},x_m,\varphi_{m+\mathrm{i}}(x_1,\ldots,x_m),\ldots,\varphi_{2N}(x_1,\ldots,x_m))$$
$$\cdot\Big[1 + \sum_{j=m+1}^{2N}\Big[\frac{\partial\varphi_j}{\partial x_m}(x_1,\ldots,x_m)\Big]^2\Big]^{1/2} dx_m \geq j - j_0\ ((x_1,\ldots,x_{m-1} \in U_1', j \geq j_0).$$

We may assume that the derivatives are uniformly bounded on U_1. Thus, there is a constant $L < \infty$, independent of j, such that

(3.3)
$$\int_{-r}^{\psi_m(x_1,\ldots,x_{m-1})-\varepsilon_j} \Phi(x_1,\ldots,x_m,\varphi_{m+1}(x_1,\ldots,x_m),\ldots,\varphi_{2N}((x_1,\ldots,x_m))dx_m$$
$$\geq (j - j_0)/L\ ((x_1,\ldots,x_m) \in U_1, j \geq j_0).$$

Write

$$\tilde{M}_j = \{(x_1,\ldots,x_m)\colon (x_1,\ldots x_{m-1}) \in U_1', -r < x_m < \psi_m(x_1,\ldots,x_{m-1}) - \varepsilon_j\}.$$

If

$$E_i = \Big(0,\ldots,0,1,0,\ldots,0,\frac{\partial\varphi_{m+1}}{\partial x_i}(x_1,\ldots,x_m),\ldots,\frac{\partial\varphi_{2N}}{\partial x_i}(x_1,\ldots,x_m)\Big)$$

with 1 at the ith entry, $1 \leq i \leq m$, and if $g_{ij}(x_1,\ldots,x_m) = E_i.E_j$, then

$|\det g_{ij}(x_1,\dots,x_m)|\geq 1$ $((x_1,\dots,x_m)\in U_1)$ so

$$\begin{aligned}
&\int_{M_j}\Phi dS\\
&=\int_{\tilde{M}_j}\Phi(x_1,\dots,x_m,\varphi_{m+1}(x_1,\dots,x_m),\dots,\varphi_{2N}(x_1,\dots,x_m)).\\
&\qquad\qquad\qquad .|\det g_{ij}(x_1,\dots,x_m)|^{1/2}dx_1\cdots dx_m\\
&\geq\int_{\tilde{M}_j}\Phi(x_1,\dots,x_m,\varphi_{m+1}(x_1,\dots,x_m),\dots,\varphi_{2N}(x_1,\dots,x_m))dx_1\cdots dx_m\\
&=\int_{U_1'}\left[\int_{-r}^{\psi_m(x_1,\dots,x_{m-1})-\varepsilon_j}\Phi(x_1,\dots,x_m)dx_m\right]dx_1\cdots dx_{m-1}\\
&\geq L^{-1}(j-j_0)\operatorname{vol}(U_1').
\end{aligned}$$

Thus

$$\int_{M\cap U\cap D}\Phi dS=\lim_{j\to\infty}\int_{M_j}\Phi dS=+\infty,$$

which implies that $\int_{M\cap D}|f|\varphi dS=+\infty$. This completes the proof. ∎

4. Proof of Lemma 2.1

To prove Lemma 2.1 we need the following lemma which strengthens [GS1, Lemma 9].

Lemma 4.1: *Let $k\geq 2$, and let $P\subset\mathbb{R}^k$ be a compact convex polyhedral body containing the origin in its interior. Let $K\subset\operatorname{Int}P$ be a compact set and let V be a neighbourhood of P. Let $F\subset bP$ be a closed, $(k-1)$-dimensional face of P. There are a compact convex polyhedral body Q, a closed $(k-1)$-dimensional face S of Q and a $\delta>0$ such that*

(i) $P\subset\operatorname{Int}Q\subset Q\subset V$,

(ii) $T(S,\delta)\subset V$,

(iii) *if H is the hyperplane containing S and if Λ is a ray emanating from a point of K and passing through F, then*

$$\Lambda\cap T(H,\delta)=\Lambda\cap T(S,\delta),$$

that is, the segment $\Lambda\cap T(H,\delta)$ is contained in $T(S,\delta)$.

We need the following simple proposition whose proof we omit.

PROPOSITION 4.1: *Let $K \subset \mathbb{R}^k$ be a compact subset of $\{x_1 < 0\}$ and let $F \subset \{x_1 = 0\}$ be a nonempty compact convex polyhedral body in $\mathbb{R}^{k-1}$ such that $0 \in \operatorname{Int} F$. Let $\tilde{K}$ be the union of all rays emanating from K and meeting F. Given $r > 1$, there is an $\varepsilon > 0$ such that $\tilde{K} \cap \{x_1 = t\} \subset \operatorname{Int}(rF) + (t, 0, \dots, 0)$ for each t, $0 < t < \varepsilon$.*

Proof of Lemma 4.1: Observe first that the condition $0 \in \operatorname{Int} P$ is needed only for the definition of $T(S,\delta)$. As $T(S,\delta)$ in our context can be described differently, assume, with no loss of generality, that $0 \in \operatorname{Int} F$, that $F \subset \{x_1 = 0\}$ and $\operatorname{Int} P \subset \{x_1 < 0\}$. Choose $r > 1$ so close to 1 that $rP \subset V$. There is a $\nu > 0$ such that $(t, 0, \dots, 0) + rP \subset V$ $(0 < t \leq \nu)$. Let $\tilde{K}$ be the union of all rays emanating from K and passing through F. Passing to a smaller ν if necessary we may, by Proposition 4.1, assume that for each t, $0 < t \leq \nu$, $\tilde{K} \cap \{x_1 = t\} \subset \operatorname{Int}(rF) + (t, 0, \dots, 0)$. Let $Q = (\nu/3, 0, \dots, 0) + rP$, let $S = (\nu/3, 0, \dots, 0) + rF$, and let $\delta = \nu/3$. Then

$$T(S,\delta) = \bigcup_{\nu/3 \leq t \leq 2\nu/3} [(t, 0, \dots, 0) + rF].$$

Now (i) and (ii) are clearly satisfied. To see that (iii) is satisfied let Λ be a ray emanating from a point in K and passing through F. Then for each $t > 0$, $\{x_1 = t\} \cap \Lambda$ is a point which, if $\nu/3 \leq t \leq 2\nu/3$, is contained in $T(S,\delta)$ by the preceding discussion. This completes the proof. ∎

PROPOSITION 4.2: *Let $\mathcal{C} \subset \mathbb{R}^k$ be a closed convex cone with vertex at the origin and let $\gamma\colon [0,\infty) \to \mathbb{R}^k$ be a C^1 path such that $\gamma(0) = 0$ and $\gamma'(t) \in \mathcal{C}$ $(t \geq 0)$. Then $\gamma(t) \in \mathcal{C}$ $(t \geq 0)$.*

Proof: Let $T > 0$. Then

$$\gamma(T) = \int_0^T \gamma'(t)dt = T.\lim \sum_{k=1}^{n} \gamma'(\xi_k)\Big[\frac{t_k - t_{k-1}}{T}\Big],$$

where $0 = t_0 < \xi_1 < t_1 < \cdots < t_{n-1} < \xi_n < t_n = T$. The sum in the bracket is a convex combination of $\gamma'(\xi_j) \in \mathcal{C}$, $1 \leq j \leq n$ so it belongs to $\mathcal{C}$. Since $\mathcal{C}$ is a closed cone it follows that $\gamma(T) \in \mathcal{C}$. This completes the proof. ∎

Proof of Lemma 2.1: Let $j \in \mathbb{N}$, $j \geq 2$, and let Φ_{ji}, $1 \leq i \leq n_j$, be the closed $(2N-1)$-dimensional faces of E_j. By Lemma 4.1 there are $\delta_j > 0$, compact polyhedral bodies P_{ji}, $1 \leq i \leq n_j$, $P_{j0} = E_j$, $P_{j,n_j+1} = E_{j+1}$, satisfying

$$\operatorname{Int} P_{j0} \subset\subset \operatorname{Int} P_{j1} \subset\subset \operatorname{Int} P_{j,n_j+1},$$

and for each i, $1 \leq i \leq n_j$, a closed $(2N-1)$-dimensional face F_{ji} of P_{ji} such that (2.1) holds and such that if Λ is a ray emanating from a point in E_{j-1} and meeting Φ_{ji} for some i, $1 \leq i \leq n_j$, and if H_{ji} is the hyperplane containing F_{ji} then $\Lambda \cap T(F_{ji}, \delta_j) = \Lambda \cap T(H_{ji}, \delta_j)$. So, if $z \in \Phi_{ji}$ for some i, $1 \leq i \leq n_j$, if V is the union of all lines passing through z and meeting E_{j-1} and if W is the component of $\operatorname{Int} V$ which misses E_{j-1} then $W \cap T(F_{ji}, \delta_j) = W \cap T(H_{ji}, \delta_j)$. In particular, $W \setminus T(F_{ji}, \delta_j)$ has two components W_1 and W_2 and any arc connecting a point in W_1 with a point in W_2 must contain a subarc λ contained in $T(F_{ij}, \delta_j)$ with endpoints in different boundary components of $T(H_{ji}, \delta_j)$, that is, in two parallel hyperplanes at the distance δ_j. Thus, the length of λ is at least δ_j.

We show that F_{ji} and δ_j have the required properties. To see this, let $\gamma: [0,1] \mapsto \mathbb{C}^N$ be a $\mathcal{C}^1$ arc, $\gamma([0,1)) \subset D$, $\gamma(1) \in bD$, such that $\gamma'(1)$ is not tangent to bD at $\gamma(1)$. Denote by $\mathbb{B}$ the open unit ball in $\mathbb{C}^N$. Given $\varepsilon > 0$, denote by $\mathcal{C}_\varepsilon$ the closed cone consisting of all rays emanating from the origin and meeting $\gamma'(1)+\varepsilon\overline{\mathbb{B}}$. Since $\gamma'(1)$ is not tangent to bD at $\gamma(1)$ one can choose a neighbourhood U of γ, $\varepsilon > 0$, $r < 1$ and $\nu_0 \in \mathbb{N}$ such that for each arc $\lambda \in U$, $\lambda([0,1)) \subset D$, $\lambda(1) \in bD$, for each t, $r \leq t \leq 1$, each ray emanating from $\lambda(t)$ and contained in $\lambda(t) + (-\mathcal{C}_{3\varepsilon})$ meets E_{ν_0} and we have $|\lambda'(t) - \gamma'(1)| < \varepsilon$. Passing to a smaller U if necessary we may assume that there is a $j_0 \in \mathbb{N}, j_0 > \nu_0$, such that $\lambda(r) \in \operatorname{Int} E_{j_0}$ for all arcs $\lambda \in U$ as above.

Let $j > j_0$ and let λ be as above. Since $\lambda(r) \in \operatorname{Int} E_{j_0}$ and $\lambda(1) \in bD$ it follows that $\lambda([0,1))$ meets bE_j, so there are t, $r < t < 1$, and i, $1 \leq i \leq n_j$, such that $\lambda(t) \in \Phi_{ji}$. Since $\lambda'(\tau) \in \mathcal{C}_\varepsilon$ $(t \leq \tau \leq 1)$ it follows by Proposition 4.2 that $\lambda(\tau) \in \lambda(t) + \mathcal{C}_\varepsilon$ $(t \leq \tau \leq 1)$, so $\lambda(\tau) \in \{\lambda(t)\} \cup \operatorname{Int}[\lambda(t) + \mathcal{C}_{3\varepsilon}]$. The preceding discussion now shows that $\lambda([t,1)) \cap T_j$ contains an arc of length at least λ_j. This completes the proof. ∎

5. Remarks

As in [J, GS2] it is easy to see that Theorem 1.1 holds if $D \subset\subset \mathbb{C}^N$ is a strictly pseudoconvex domain with $\mathcal{C}^2$ boundary.

When proving Theorem 1.1 we actually proved that

$$\int_{M\cap D} \varphi \max \operatorname{Re} f, 0\} dS = +\infty. \tag{5.1}$$

Thus, in Theorem 1.1, (1.1) could be replaced by (5.1).

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